Name:

Fall 2017 Math 245 Exam 1

Please read the following directions:
Please write legibly, with plenty of white space. Please print your name on the designated line, similarly to your quizzes (last name(s) in ALL CAPS). Please fit your answers in the designated areas. To get credit, you must also show adequate work to justify your answers. If unsure, show the work. All problems are worth 5-10 points. The use of notes, calculators, or other materials on this exam is strictly prohibited. This exam will begin at 1:00 and will last at most 50 minutes; pace yourself accordingly. Please leave **only** at one of the designated times: 1:20pm, 1:40pm, or 1:50pm. At all other times please stay in your seat (emergencies excepted), to ensure a quiet test environment for others. Good luck!

Problem	Min Score	Your Score	Max Score
1.	5		10
2.	5		10
3.	5		10
4.	5		10
5.	5		10
6.	5		10
7.	5		10
8.	5		10
9.	5		10
10.	5		10
Exam Total:	50		100
Quiz Ave:	50		100
Overall:	50		100

Problem 1. Carefully define the following terms:

a. $\leq$ (for integers, as defined in Chapter 1)

b. factorial

c. Associativity theorem (for propositions)

d. Distributivity theorem (for propositions)

Problem 2. Carefully define the following terms:

a. Addition semantic theorem

b. Contrapositive Proof theorem

c. Direct Proof theorem

d. converse

Problem 3. Let a, b be odd. Prove that $4a - 3b$ is odd.

Problem 4. Suppose that $a|b$. Prove that $a|(4a - 3b)$.

Problem 5. Simplify $\neg((p \to q) \vee (p \to r))$ to use only $\neg, \vee, \wedge$, and to have only basic propositions negated.

Problem 6. Without truth tables, prove the Constructive Dilemma theorem, which states: Let p, q, r, s be propositions. $p \to q, r \to s, p \vee r \vdash q \vee s$.

Problem 7. State the Conditional Interpretation theorem, and prove it using truth tables.

Problem 8. Let $x \in \mathbb{R}$. Suppose that $\lfloor x \rfloor = \lceil x \rceil$. Prove that $x \in \mathbb{Z}$.

Problem 9. Prove or disprove: For arbitrary propositions p, q, $(p \downarrow q) \to (p \uparrow q)$ is a tautology.

Problem 10. Prove or disprove: For arbitrary $x \in \mathbb{R}$, if x is irrational then $2x - 1$ is irrational.